# A Computational Study of the Unsteady Shock-Wave Structure in a Two-Dimensional Transonic Rotor

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## Abstract

SOME of the loss mechanisms associated with unsteady transonic flow in a two-dimensional cascade are discussed. For this purpose, the unsteady flowfield generated by a shifting inlet velocity profile modeling approximately the stator wake as a linear velocity profile is compared to the steady field created by a uniform inlet. Both flows are generated by a numerical solution of the Euler equations using the MacCormack difference scheme. The results indicate that there is considerable influence of the inlet flow unsteadiness on the location of the shock wave that moves upstream for a distance on the order of 15% of chord length. The shock also tends to oscillate with an amplitude 5% of chord length. These shock oscillations generate pressure waves propagating downstream of the rotor increasing the flow mixing losses. They should also affect the boundary layer detrimentally. An overall effect of the shifting velocity deficit is a 10% of steady-state lift.

## Contents

### Introduction and Analysis

Recently, considerable experimental evidence has shown that unsteady flow effects play a significant role as a loss-generating mechanism. Following Kerrebrock,[1] the steady-state calculations of the viscous and shock-wave losses in a transonic rotor do not agree with the measured efficiencies that are considerably lower than expected. In general, transonic rotors with a lower level of unsteadiness present a higher efficiency. In addition, as reported by Weyer and Hungenberg,[2] in a given rotor, losses are concentrated mainly in the regions of the blade where the maximum values of rms fluctuations from the steady flow in rotor coordinates are measured, i.e., at the hub and tip of the blading.

Presently available state-of-the-art transonic rotor design methods are based on a three-dimensional inviscid, steady-state computation with realistic blade, hub, and shroud geometric description, that is later corrected to account for the viscous effects.[1] The introduction of unsteadiness in the design procedure complicates the whole design process immensely. The flowfield in a turbomachine rotor is virtually unsteady due to the presence of the stator blades that affect the rotor dynamically through the action of the stator pressure field, kinematically altering the inlet velocity profile. An additional complexity is the lack of periodicity of the field between two consecutive blades caused by the nonequal number of rotor and stator blades. However, it should be clear that if an exact solution for the transonic rotor flowfield is demanded incorporating the three-dimensional geometry, the compressibility, the viscous, and the nonperiodic effects, one should have to wait for the supercomputers of the future. With the present computer capabilities, a rough estimate of the unsteady flow effects can be attempted with relatively simple means if we relax the nonperiodic requirements and examine a "periodic stage," i.e., a stage with an equal number of rotor and stator blades. This approach is followed herein.

The purpose of this work is to study the effects of a distorted inlet flow on the shock structure, with the hope of identifying the main loss mechanisms associated with the unsteadiness in transonic rotors in the absence of viscous effects. To that effect, a simplified model of the stator influence on the rotor inlet flow is examined, simulating the stator wake as a velocity deficit. To investigate the effects of this inlet flow

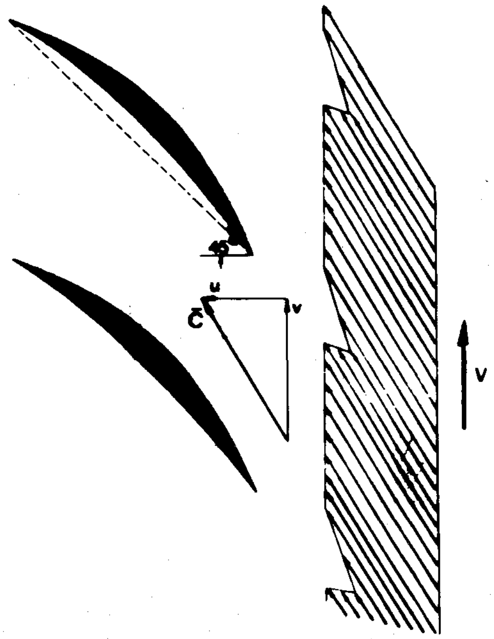


Fig. 1 Cascade geometry and shifting velocity profile.

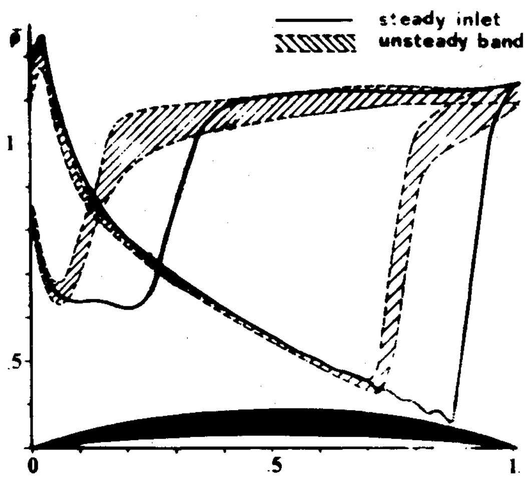


Fig. 2 Density distribution comparison between steady and unsteady flows on the blade.

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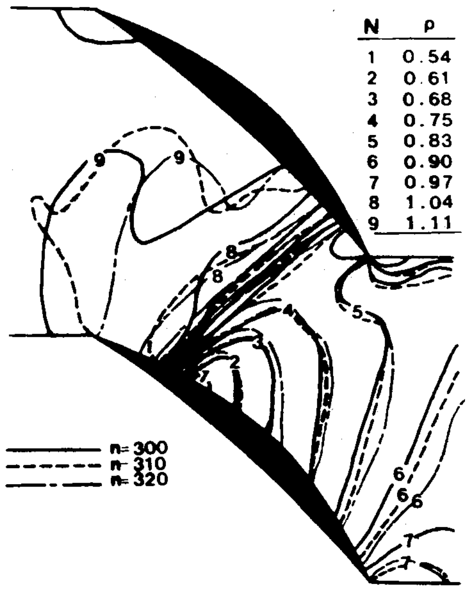


**Fig. 3 Unsteady density field within the cascade.**

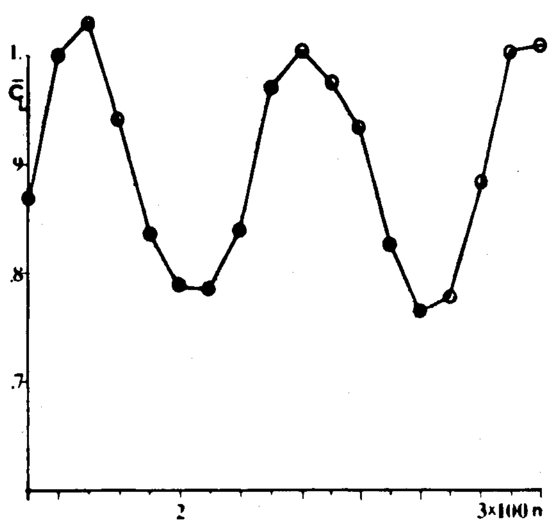


**Fig. 4 Unsteady blade lift variation.**

distortion, the well-known MacCormack difference scheme is used to solve the Euler equations in conservation law form

$$U_t + F_x + G_y = 0 \text{ with } p \sim \rho^V$$

in a two-dimensional cascade. The blade used is formed by two circular arcs with radii $R_1 = 1.5c$ and $R_2 = 5c$, where $c$ is the blade chord length, at a stagger angle $\xi = 45$ deg with a solidity $s/c = 1$ (see Fig.1).

For the purpose of comparison two flowfields were computed: one having a uniform inlet giving a steady flow, and an unsteady flowfield generated by a nonuniform inlet profile. The relative inlet Mach number is $M = 0.8745$, and the stage pressure ratio $p_{ex}/p_{in} = 1.14$. For the generation of the unsteady flow, the velocity profile indicated in Fig. 1 is shifted for a distance $\Delta y$ equal to the mesh spacing in the direction of rotation after an interval $\Delta t = \Delta y/v$, where $v$ is the rotational velocity of the rotor. The profile has a linear velocity distribution ranging from a minimum value $M_{min} = 0.656$ to the free-stream value of $M_{max} = 0.8745$, where the pressure is assumed constant. This profile creates a mass flow reduction on the order of 4.6%, a momentum reduction of 8.4%, and a kinetic energy reduction of 11.5%.

## Results and Conclusions

The computational mesh consists of $70 \times 40$ mesh points. The steady-state solution yielded the following density distribution on the blade surfaces, as presented in Fig. 2. It is observed that there is a shock extending from the pressure to the suction side, where the Mach numbers are 1.3 and 1.9, respectively. The analogous results for the unsteady inlet case are also presented in Fig. 2 for comparison, in the form of a band covering the resulting density oscillations. Notice the shock oscillation with an amplitude of 5% of chord length around the mean position that is approximately 15% of the chord length upstream of the steady shock position. This upstream displacement of the shock must be attributed to the reduction of the mass flow caused by the presence of the inlet wake. Similarly, the shock oscillations should be generated by the periodic impingement of the retarded flow in the wake.

The general unsteady structure of the flowfield is illustrated in Fig. 3; i.e., the contour plot of the density within the cascade, corresponding to different times, after the establishment of periodic flow conditions at $n = 250$ computational cycles. From this plot, it is relatively clear that the pressure field downstream of the shock is very disturbed compared to the upstream pressure field. This indicates that the shock oscillations generate substantial pressure fluctuations downstream of the shock that cannot propagate upstream due to law of forbidden signals of supersonic flows.

A global effect of the unsteadiness is the fluctuation of the blade lift, as indicated by the plot of the unsteady lift as a fraction of the steady lift (see Fig. 4). The general shape of the curve that exhibits a rather abrupt rise followed by a more gradual decrease should be attributed to the nonlinearity of pressure wave propagation. The lift oscillations caused by the shock displacements are also related to the nonuniformities of inlet flow vorticity. Similar results have been reported by Barr[3] from measurements of the stagnation pressure downstream of turbomachine rotors for $M < 1$. Finally, the observed effect of disturbance attenuation agrees with the analytic results presented by Savell and Wells.[4]

The results presented herein indicate that there is considerable influence of the unsteadiness on the structure of the flowfield of a transonic rotor. The main effect appears to be a significant upstream displacement of the shock wave that tends to oscillate. These oscillations should increase the flow losses mainly by their effect on the boundary layer, as well as by their action as a pressure wave source considerably increasing the mixing losses downstream of the shock. Therefore, a plausible method for the reduction of the rotor level of unsteadiness is to increase the relatively steady portion of the blade by moving the shock wave further downstream. The analysis of the rotor response to inlet flow distortions at a supersonic Mach number is an interesting problem currently under consideration. The increase of the relative Mach number to supersonic values would move the shock downstream, increasing the supersonic region on the blade. However, it would also increase the steady flow losses at the shock. At this point, the conflicting nature of these phenomena makes it difficult to predict the net effect of the Mach number on the losses prior to the calculation of the boundary layer and mixing losses of the flow.

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